Spring '26

Nevada State University

The Desert Mathologer

Program Newsletter for Mathematics + Data Science

Cover photo: Photo by Sungju Moon, *Stargazing Night, Feb 17, 2026* (NS Math & Data Science Club).

In This Issue

HAPPY SPRING SEMESTER, SCORPIONS!

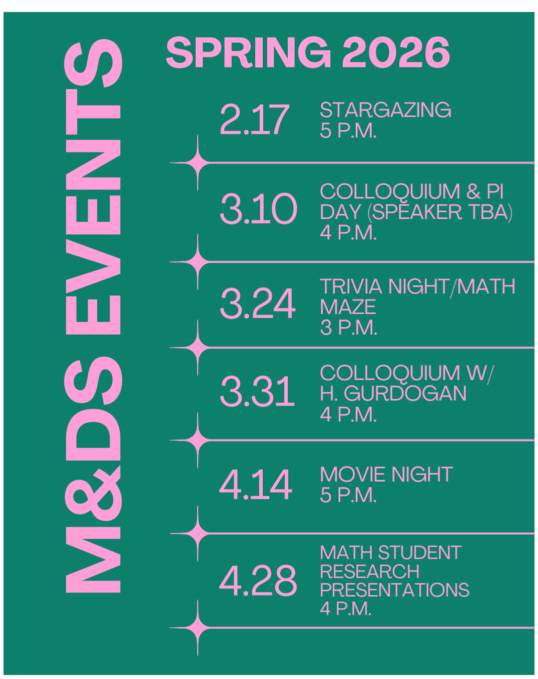
STAYING motivated and focused in STEM classes is no easy task. I can speak from experience when I say that learning mathematics, statistics, or any data-adjacent field can be intimidating, and struggling alone should never be the solution!

The Math and Data Science Club creates a welcoming environment where students from STEM (and non-STEM!) disciplines can come together to connect, collaborate, and support one another through challenging times. Learning is not always linear (pun intended), so having a community to lean on makes all the difference. We host monthly club events where students from all disciplines can have fun and step away from the pressure of classes and assignments, as well as monthly colloquiums where we gather to learn about new and exciting topics. We're also looking forward to celebrating Pi Day this March!

We hope you'll join us for **Tuesday Tea** in the Dawson Lobby at 3:00pm, or see you at one of our upcoming club events!

Stay updated by following our club on Instagram: `@mnd.nsu`

Jules Perna
President
Math & Data Science Club



This semester's newsletter was compiled by Dr. Sungju Moon with the help of many. Special thanks to all contributors and proofreaders.

News & Events

- The **All Math and Data Science Advising Meeting** is scheduled for Thu, Mar 26, 3:30–5PM in Dawson 108. This is your opportunity to ask questions regarding your degree program, and network with faculty and peers. Please RSVP at `https://nevadasc.co1.qualtrics.com/jfe/form/SV_b4bPKWyEshviJj8`

- The **Putnam Exam** was held Saturday, December 6th, 2025.

  Join your fellow math students for the annual Putnam exam this upcoming December! MATH 491 is running currently on Thursdays at 2–3:20PM (stop by!) and is on the books for next semester on **Mondays at 3:30–4:50PM**. More on Page 6.

- **MathRocks!** is a monthly math contest for students in grades 1–12.

  Recent high achievers list:

  - Abdulla A. (4th place in Nevada, September 2025)

  The 2026 State Championship will be held on **Saturday, April 11**. We are looking for student volunteers. Contact Dr. Aaron Wong if you want to get involved.

- The **Math and Data Science Club** has plans!

  - *Tuesday Tea* continues strong each week in the Dawson Lobby. Enjoy tea, coffee, and light snacks. Bring your favorite math mug! It's a chance to relax, chat about math or life, play board games, or work on homework alongside friends from math and data science.
  - *Colloquium Talks* are held on a quasi-monthly basis immediately following tea. In January, we welcomed Dr. Han-Gyul Jin (Pusan National University), who spoke about numerical weather prediction. Our π day special speaker was Dr.^{2} Pushkin Kachroo on the topic of infinities and zeroes. We look forward to welcoming Dr. Hubeyb Gurdogan from UCLA on Mar 31. Stay tuned for student talks on Apr 21 (algebra special topics) and Apr 28 (CURM MATH 498 students).
  - Earlier this semester, Dr. Chad Curtis led *Stargazing Night* featured on the cover image. Stay tuned for *Trivia Night* on Mar 24 and *Movie Night* on Apr 14. The tentative movie choice is *Mean Girls* (the classic version).
  - Stay up to date by following the club Instagram handle `@mnd.nsu`

NEVADA STATE UNIVERSITY

MATH AND DATA SCIENCE ADVISING MEETING

THIS IS YOUR OPPORTUNITY TO ASK QUESTIONS REGARDING YOUR DEGREE PROGRAM AND NETWORK WITH FACULTY AND PEERS.

THURSDAY, MARCH 26

3:30 PM-5:00 PM

Dawson 108

RSVP: https://nevadasc.co1.qualtrics.com/jfe/form/SV_b4bPKWyEshviJj8

For more information contact, Dr. Krystle Oates
Krystle.oates@nevadastate.edu

Dr. Chad Curtis demonstrates the telescope for the *Stargazing Night* event.

Fall 2026
Course Highlights

MATH 314 History of Mathematics

TuTh 11:00–12:20PM

MATH 314 offers a sweeping overview of mathematics from the historical point of view (and an explanation as to why it turns out topology is the final boss). An oasis for the *Desert Mathologer*: take a long draw, freely discuss, and explore a mission-driven course filled with games and mini-activities of your choosing. Academic benefits: a broader view of mathematics as a whole, plus practice in scientific communication. Side benefits: collect mathematician cards like Pokémon!

MATH 491 Problem Solving Workshop

Mondays 3:30–4:50PM

"If you don't feel stupid doing math, you aren't doing math hard enough." Come think about interesting and hard math problems. Show up every week to pass the class. Do it for three semesters and it counts as an upper division math elective. No prerequisites. Open to anyone willing to feel stupid doing math. (S/U Grading)

DATA 320 Mathematical Modeling

Meeting times TBA

"It's the mathematical modeling course. That's all I have."

—Dr. Chad Curtis

INTERNSHIP OPPORTUNITY: Apply today for tuition coverage for a data analytics training opportunity with *Harnessing the Data Revolution for Fire Science* (HDRFS). **The application deadline is March 27, 2026.** Visit `https://hdrfs.epscorspo.nevada.edu/funding-opportunity/2026-tuition-covered-data-analytics-training/`

A Slice of π Day

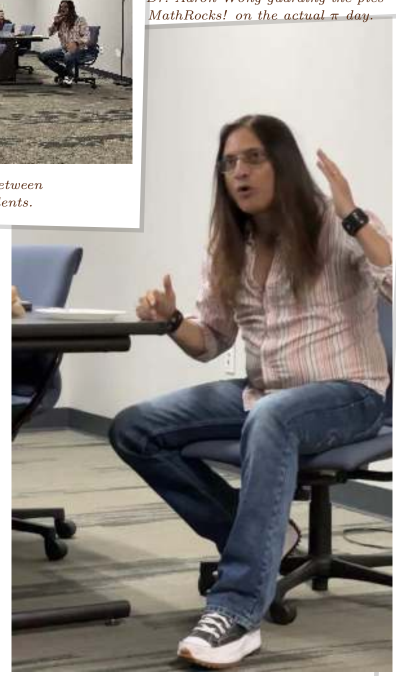
Photo scraps: Mar 10–14, 2026

Pi Day would still be Pi Day without the pies

Dr. Aaron Wong guarding the pies MathRocks! on the actual π day.

A spirited discussion between Dr. Kachroo and students.

Colloquium talk about infinities captivating the audience.



A lively Q&A session following the talk.

2025-26 NEVADA MATH

MathRocks!

CONTEST SERIES

ABOUT

MathRocks! is for students grades 1-12. For more information, please visit:
nevadamath.org/mathrocks

Series (6 contests): $120
Single contest (early): $25
Single contest (regular): $30

For 2025-26, we will be incorporating the following national contests:

- American Scholastic Mathematics Association
- MathLeague.com
- Mathematical Olympiads for Elementary and Middle Schools
- Pi Math Contest
- Primary Math Challenge

CONTEST DATES

tentative dates

Contests run: 9am-12:15pm
Check-in: 8:45 am

2025

- September 20
- October 25
- November 22

2026

- January 24
- February 21
- March 14

2026 State of Nevada Math Championship: April 11

INFO

Sponsors Appreciated & Volunteers Needed!

Financial assistance available. Please visit:

nevadamath.org

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Please email questions to Angela@NNVMath.org

Nevada Math is a 501(c)3 non-profit Charitable Organization.

PUMPKIN REFLECTIONS

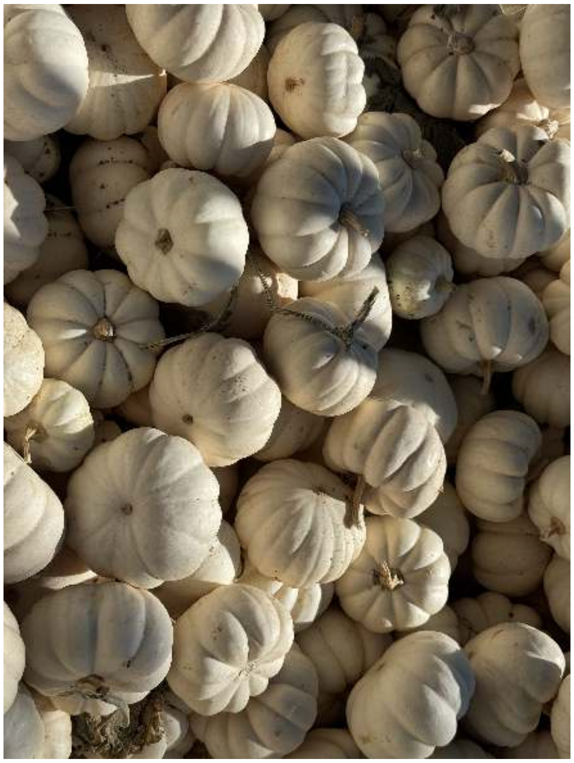
In October 2025, the Math & Data Science Club celebrated Fall by arranging a trip to Gilcrease Orchard, followed by pumpkin painting and a movie night on campus. The following documents the day in photos and in the words of those who were there.

The orchards had me turned around, but math people have a way of finding each other. Once we did, the path forward was obvious. I was lost in thought, lost in math, and lost among the orchards, that is, until I found my people. Math people. Reunited, we found the way.

It did get a bit scary when the first apple I picked had about 4–5 baby spiders on it.
I enjoyed seeing all the pumpkins that hadn't been cut yet.

The pumpkin painting was another experience where I was able to connect with some of my peers. You do not get many opportunities to just talk about nonsense during the semester, so it was a nice couple of hours to relax and socialize.

I got to check off my yearly visit to Gilcrease and enjoyed the fact that I was able to enjoy this experience with similar minded individuals. The apple cider juice and donuts were amazing.



Painting pumpkins with friends and classmates. Being around friendly animals. Picking fruits like pomegranates from the orchard. This event was highly therapeutic. After attending, I noticed a greater ability to give my classes additional focus and enthusiasm.

There is something that happens when you step outside the campus together. The formulas fade for a moment, and what remains is simply people. Connected, supported, and a little more rooted, like the pumpkins on the vine.

Math & Data Science Club Presents...

JEOPARDY!

TRIVIA NIGHT

Categories spanning **History**, **Quick Thinking**, **Clever Math Tricks**, and more!

200	200	200	200	200	200
400	400	400	400	400	400
600	600	600	600	600	600

EVENT DETAILS

Tuesday, March 24, 2026
Math Tea at 3 PM · Game Starts 3:45 PM
Dawson 106

Open to all majors · Sandwiches to be provided
Special item awarded to the winning team (*NO* cash prizes)

*** COME AS INDIVIDUALS OR FORM A TEAM UP TO 3! ***

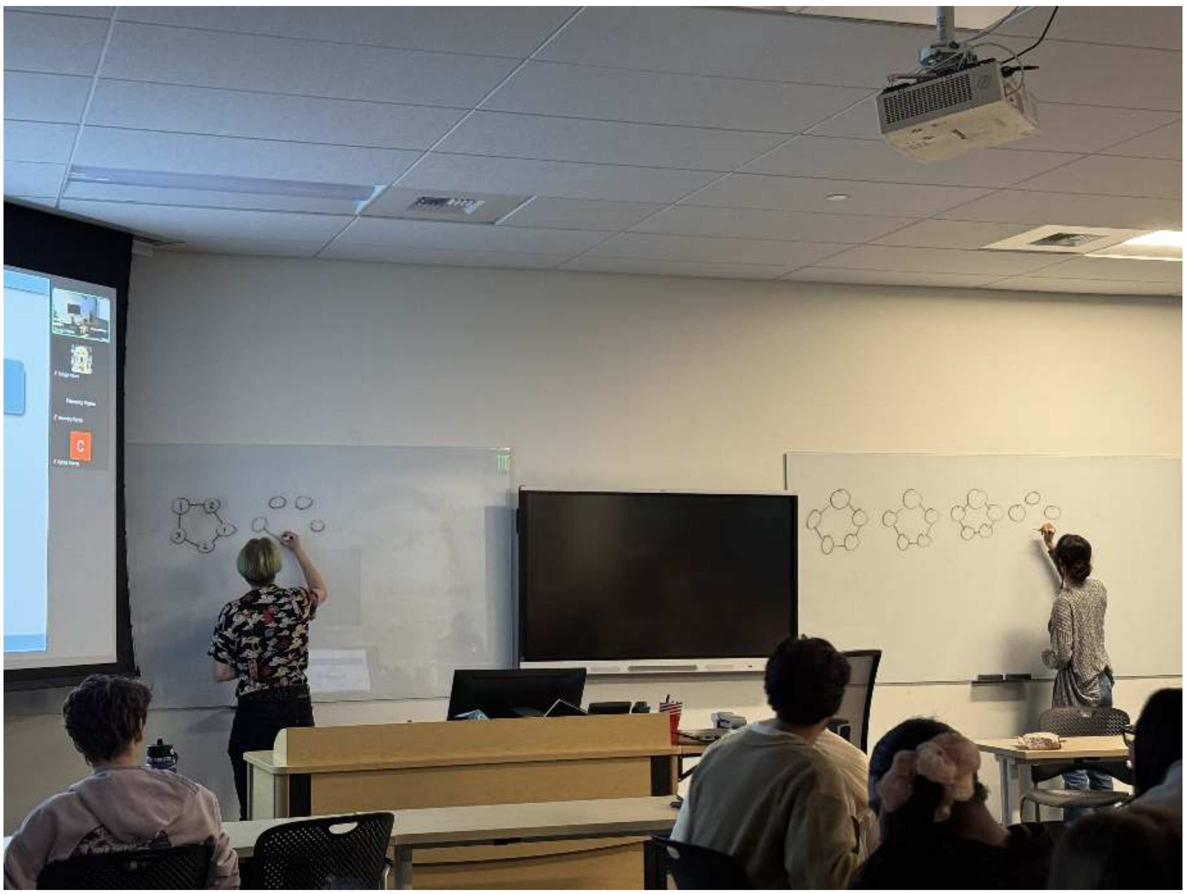
Please RSVP

Sandwiches provided for registered participants only
Questions? Email MDS Club President or Dr. Moon
JULIANNE.PERNA@STUDENTS.NEVADASTATE.EDU
SUNGJU.MOON@NEVADASTATE.EDU

IN THEIR OWN WORDS

The Capstone Experience in Mathematics

by Carolina Acero & Amber French



CAPSTONE was a really interesting course because it differed a lot from how other courses are run at Nevada State. The class started similarly to other courses when Dr. Sahajpal introduced us to the topic we were going to research, a branch of graph theory called Zero Forcing. As we became more familiar with Zero Forcing, the class shifted to us sharing our findings and thoughts with the class every time we met. We then started looking more into different sub-categories of Zero Forcing, and once we found the sub-category we were interested in, we split into different research groups. Our group focused on Multi-Color Zero Forcing, which introduced looking at graphs with more than two colors. Once split into groups, we no longer met as a class but had separate group meetings with Dr. Sahajpal. These meetings felt much more like a collaboration where we all shared our thoughts and findings each week and discussed what to look at next.

For our research on Multi-Color Zero Forcing, we studied how the deletion of an edge in 3-, 4-, and 5-vertex cycles affects the number of forcing steps, the number of propagating forcing steps, and the end state of a graph in the multi-color forcing procedure with the 3-cyclic forcing network, and studied the changes in the data to find patterns and produce solid results. We exhausted every initial coloring of 3-, 4-, and 5-vertex cycles in which each color 1, 2, and 3 appears at least once (excluding any initial colorings obtained by rotations and reflections of other initial colorings). Over the semester, we considered and worked through 150 distinct cases, looking for patterns. Doing these calculations was very repetitive and tedious, and it took us several weeks to get through all possible cases. We worked on this research in class, at home, and even at work during breaks (scribbling on napkins!). From our research, we were able to formulate conjectures and one proven theorem.

"In Capstone, you research areas that still have so much more to discover. You start by looking at what's out there, seeing what questions others have about the topic, and trying to see if you can find the answers to these questions."

By the end of the semester, we put all our research together, explaining the process, the patterns we found, the conjectures we made, and what we were able to prove. To share our research, we created a presentation that would be shared during a colloquium for the Math and Data Science Club, and we wrote a more in-depth paper describing our methods, findings, and already existing research. To prepare for our presentation, we first had a mock presentation for our class, giving us a chance to see what should be changed, what should be added, and what may not be necessary. We both also practiced independently at home to ensure we were comfortable presenting. I (Amber) happened to get sick around the time of the presentation, and practiced the presentation so much that I lost my voice! I ended up doing half the presentation whispering to a group of 30 people. Everyone was respectful as we presented and remained quiet enough so everyone could hear me.

We really enjoyed taking *Capstone* because the experience is so different from the experience you get from taking other math courses. In other math courses, you're given specific problems to solve, knowing the solutions you're working for. There are always deadlines, and you're rewriting proofs that have been done hundreds of times before. In *Capstone*, you research areas that still have so much more to discover. You start by looking at what's out there, seeing what questions others have about the topic, and trying to see if you can find the answers to these questions. You also get to explore how things work, try to spot your own patterns, and try to prove your own findings. It's exciting knowing that the research we've built over this semester is brand new research, and can be used for further analysis by other mathematicians. *Capstone* is both exciting and scary because you go into it not knowing what you'll find or if you'll find anything, but what's important is that you put the time in and explore what interests you. ■

LEARN MORE

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RONALD E. MCNAIR POST-BACCALAUREATE ACHIEVEMENT PROGRAM

EXIT INTERVIEWS

We sat down with two graduating mathematics majors to reflect on their journeys. They offered candid perspectives on the math program, their academic experiences, and the community. Their insights as well as some practical advice for students follow below:

Harmony Styron

Please introduce yourself.

Hi, my name is harmony Styron and I'm graduating this fall. My major is mathematics. I could talk about what I majored in before: I was at UNLV. I used to major in computer science, and I realized that I was more interested in the theory behind a lot of the math that they were talking about, and I was interested in essentially understanding the way things work. And so I guess physics, in a way, got me interested in math. And I switched to physics for one semester, and then I left.

Looking back, how has your view of math changed since you began your studies?

I used to have this idea that if you didn't have the talent for math, you couldn't do it. And so I heard lots of people say, Oh, I'm just bad at math. I don't know math. I'm never going to be good at it. And I think at one point, when I was really young, I thought I was one of those people. But when you actually get into practice for math, and you look into the concepts, and you get excited about what that means, and maybe make connections to the real world, you know, modeling it, applying it to things, it really changes your view. And that is one of the major changes that happened to me. I mean, when I was younger, I scored pretty low in mass standardized tests, and then I just decided, What if I do like math? And then I tried it, and I loved it.

I guess you kind of already answered it in a way. Was there some a moment when the math clicked? Was it like a specific moment, or was it a gradual process?

I used to watch these YouTubers, I guess Vsauce is one of them, a long, long time ago, and he would talk about these very interesting theorems and just make it accessible to me when I was younger. And I think being exposed to the cool theorems of math. You know, I think of what we were doing recently, knot theory. Although I was aware of it in the past, I still think it is really interesting that you can actually have a field of math that intersects with art and also has applications. That really made me excited to learn more.

This is getting more practical: what skills or habits of mind or any kind of techniques you have developed as a math major?

One of the main things that I found learning about proofs is that it's a completely different way of thinking than just making connections and seeing, you know, trying to see what sticks, throwing stuff at the wall and trying to see what sticks, is kind of how I thought before. But proofs, you actually have to think very carefully, and you have to think and think and think and think in order to actually find a path for a proof. And I was not used to it. It's almost like a switch in my brain: when I switched to that type of thinking, it became really exciting, and I found myself feeling more knowledgeable, and I also felt like I was connected with a world that I previously hadn't been. So it was really cool to learn that way of thinking.

Elaborate a little more. What did you find exciting? The newness of it?

Yes, well, it's a new thing. Because when you prove something, you know that it's true. It's 100% true, if your argument is sound. And that was kind of crazy to me because, before I was kind of under the impression that you could try and make certain connections, and then you would say, you know, in my view, this is true, my view this is, but in math, it's completely different. You actually get clarity, you get precision, and you get a rigorous frame of mind, right? You're not relying on some kind of authority figure to tell you this is a good, this is a pretty, this is valuable.

"I used to have this idea that if you didn't have the talent for math, you couldn't do it. Then I just decided, What if I do like math?"

That's a cool perspective to have. Switching gears, do you have any memorable moments throughout your time here in the math program? I am aware you have just talked about knot theory just now.

Yes, knot theory is one of them, but I doing independent studies such as Undergraduate Research on top of *Capstone* It exposed me to a different style of learning. There's a traditional learning style, I guess, and then there's a way of learning where you actually are interactive, and you do projects and you do presentations, and read papers on your own. I was not used to that, but I realized that I actually learn a lot more if I I get outside of that traditional learning model and follow that independent and ressearch-oriented learning.

So, all of this was really memorable for me and impacted my learning. My favorite course, if I have to pick, would be my Undergraduate Research course where I studied catastrophe theory. When it comes to a more traditional course, I would pick *Real Analysis*. It was a really cool class because I learned a lot about the the rigorous backing for calculus. And when you first start calculus, you just plug and chug. You just you don't know where it's coming from. You don't know what it means.

Well, in my calculus class, I do try to explain that...

Well, many students don't pay attention to any of that. But with real analysis, you come at it from a different way. You come at it with that same rigorous thinking style that I was talking about earlier, and that's really cool.

Now this question is about relationships. Did you make friends while you were here? Any memorable interactions or activities you recall?

Okay, I might get a little personal here. I struggled with school and picking the right college for me. At some universities I attended, there was a sense that I didn't really feel welcome and I didn't feel like I was in the right place. But when I went to Nevada State University, everyone was so kind to me, and it was so different from how I felt about universities before, maybe because it has smaller class sizes. It is easy to feel like you're just in a crowd. The instructors are overworked in the larger classes. Here, the instructors were so kind, the faculty were so kind, and then also the students were kind as well. I did presentations with Carlos and Mary and. They were really cool to work with. I was really appreciative of the fact that I had this opportunity and that I found this place. I don't think, I don't think I would have been able to finish college without that support system.

Well, I am glad you found us! As you know, the math cohort is a very small communty

So, I guess it makes sense to help each other.

Speaking of the community, what would you say to future math majors, that is, people who are about to go into math?

I would say, there are times where you might think, "well, this is very abstract. It's very overwhelming." Then you can just realize through practice and through trying that you can actually appreciate math, and you can also appreciate the applications of math, and it's really worth it. I would just say, Do it? Do a math major. It's so worth it, and you learn so much.

Finally, where do you go from here?

Well, I'm already interested in doing furthering what my undergraduate research was on through MATLAB, and I'll be doing that in the very immediate future. Then for the far future, I just want to learn more about research, and I'm probably going to look for postgraduate opportunities in math, and maybe also with a bit of a physics concentration as well. And I'm very interested in understanding how the world works, and I think that research can help me achieve that.

Carlos Chavez

To begin, could you introduce yourself and share a bit about your academic background and activities you like to do for fun?

Hello, my name is Carlos Chavez. I'm a dual major in secondary education and mathematics. Outside of math, I like to read, I recently started playing bass guitar, and I used to skateboard.

Looking back, what initially drew you to mathematics, and what influenced your decision to pursue it as a major?

I felt that math was concrete. There was no second guessing—as long as you used everything correctly, the answer was always going to be the answer. There are different ways to get to the same answer, but the answer itself doesn't change, and I always liked that about math. That sense of certainty was something I found really appealing early on.

I was also good at math, which was the main reason I chose it. My math teachers growing up were amazing, especially since middle school. They were always happy and cheerful. I remember my 7th grade teacher would dance and make songs, and I felt like they always tried the hardest. Other than science teachers, they were the ones who tried the hardest for me.

Harmony and Carlos
during Capstone presentation

You mentioned that you were drawn to mathematics because of its concreteness. How did that idea influence your decision to pursue mathematics more seriously? Has your perspective on mathematics changed since then?

When I first started, I thought math was a system of just getting from A to B. Which it is. But after taking more classes, I started to see the logic behind that system.

The logic and problem solving in math is different from something like physics. In physics, you can actually see what's going on, but in math, you really have to think about the logic more concretely.

My view has changed from seeing math as just equations and procedures to something more logical, which requires thinking more abstractly. I also see it as more creative and even philosophical.

As your understanding of mathematics became more focused on logic and reasoning, was there a particular moment or experience when things began to "click" for you?

I would say real analysis, especially at the end of *Real Analysis I*. I was amazed that using a small number of axioms, we were able to create and make sense of the real number line.

It surprised me that from just a few principles, we can essentially create numbers, and everything we do can be traced back to those axioms.

What mathematical ideas or concepts do you find yourself thinking about outside of class?

I go back to real analysis a lot, especially the axioms and how they define things. I think about ideas like what it means for something to be bigger and about inequalities.

I also really like the Cauchy convergence criterion. It makes sense—if the distance keeps getting smaller, you keep approaching a point.

I think a lot of what I think about depends on the semester. For example, during real analysis, I found myself thinking a lot about convergence. I also really enjoy the visual aspect of math, which is why I liked topology, even though it was challenging.

I also listen to a podcast called *My Favorite Theorem*, and it exposes me to different areas of math. It makes me curious about all math and reminds me that there is so much more to explore.

You mentioned enjoying the visual side of mathematics. Were there any specific courses or projects that stood out to you as especially meaningful?

"The more I've learned, the more I realize there's so much more that I don't understand yet"

The knots project was really fun. I'm glad I had the opportunity to study knot theory, because I had heard about it but never really explored it.

When I was able to manipulate a knot and recognize it as an unknot, I felt proud of myself. I have a hard time visualizing things like that, so being able to do that felt like progress.

What skills or habits did you have to develop as a math major?

The biggest thing is learning to understand instead of memorize.

At the beginning, you're given formulas and told to apply them. But in higher math, you have to understand what's going on in proofs and recognize the techniques being used, because you'll need those techniques later.

I think this starts in high school, where you're trained to memorize, and it doesn't really change until you take a proofs class. That's when you realize you can't just plug things into formulas anymore.

It's not really memorizing—it's recognizing patterns and extending ideas.

My biggest advice is: learn how to read a proof, and take *Proofs* early.

"You lift weights, but you don't actually use those movements in a game. You do it to get stronger. That's what math does—it strengthens your brain and your critical thinking."

As you were developing this way of thinking, how did your interactions with classmates and faculty shape your experience?

My classmates were amazing. At first, I was a little shy, but I got over it pretty quickly. I made friends and partners, and I would often ask them how they approached problems and how they thought about math.

Even up through later classes, I kept asking people what their process was and how they thought about things. That helped me a lot.

It also helped to see that sometimes they didn't know either. That made it easier to keep going and contribute when I could.

All of my professors have been great. I've learned a lot from them, and I've talked to them about my concerns about my ability.

You mentioned talking with your professors about your concerns regarding your ability in mathematics. How did those interactions shape your confidence and growth as a student?

They've always been encouraging, and that really helped me start thinking of myself as being better at math. Even if I wasn't always doing well on exams, I felt like my understanding of the material improved a lot, and the content became much easier to understand.

I think people should talk to their professors more. They're there to help you.

Based on these experiences, what advice would you offer to students who are just beginning their journey in mathematics?

Talk to your professors. Learn how to read proofs early. Focus on understanding instead of memorizing.

Also, don't be discouraged if things feel difficult at first. Once you start understanding the ideas behind the math, things become clearer.

How do you see mathematics shaping your future?

I want to be a math teacher. It's between that and becoming an actuary. Either way, I'll be doing math. Math is everything.

At first, I didn't really know what I would do with math. I was originally a physics major, but I didn't like chemistry, and I realized I really liked the math in physics. That led me to switch to math.

I chose the teaching track because I wanted to be a better teacher and help students understand math better, not just memorize it.

I'm planning to possibly teach while also preparing for actuarial exams, and then decide from there.

What would you say to someone who asks, "When will I ever use math?"

You use math every day—phones, TVs, earbuds, everything. Maybe not directly, but indirectly.

But more importantly, math develops your thinking. I feel like my critical thinking has improved a lot. It might also be because I'm growing as an adult, but math has definitely helped with that.

I like to explain it using sports. You lift weights, but you don't actually use those movements in a game. You do it to get stronger.

That's what math does—it strengthens your brain and your critical thinking.

Do you think math is discovered or created?

I think we're discovering it. We create tools, but the math itself is already there—we're uncovering it.

Looking ahead, what excites you most about continuing your journey in mathematics?

The more I've learned, the more I realize there's so much more that I don't understand yet. There's always more to learn, and you can never know all of math.

That's something I find exciting. ■

Alumni Updates. M.G. ('26) has been accepted to a fully funded PhD program in mathematics to start in Fall 2026. Josh Miller ('25) started a new position at Morgan Stanley and simultaneously started his master's program (OMSA) at Georgia Tech. His capstone research work has been published in the PUMP Journal of Undergraduate Research. Find it at `https://journals.calstate.edu/pump/article/view/6010/5304`

Faculty Spotlight: Dr. Tim Malacarne

This semester's newsletter faculty spotlight features Dr. Tim Malacarne, one of the regulars at Tuesday Tea and a founding member of the data science program here at Nevada State. As a math program newsletter, we were curious to hear his perspective on where the two fields meet.

Even though you are a familiar face around here, let's start from the beginning: who are you and what do you do here?

My name is Tim Malacarne. I'm an Associate Professor of Data Science. This is my 6th year at Nevada State. I'm interested in most things data related, but my research focuses on education, social networks, and popular culture's influence on society.

My understanding is that your background spans many fields—sociology, international relations—how did you get into teaching data science?

"This is the million dollar question right now. I think it will become even more about your ability to define a question, rather than your ability to get a model to run without errors."

I went to an international relations school as an undergraduate. I took no math courses, but lots of language and international politics. I quickly learned that I wouldn't be a good bureaucrat, so the Foreign Service was out. I went to graduate school to be a cultural sociologist, had to take statistics as part of my course work, really liked it, kept taking more classes in the area, and eventually got into more technical areas like social network analysis and game theory. My first job started as a sociology job where I taught the quantitative methods courses, but they wanted someone to teach data science. I said I wasn't interested. They failed to hire someone else and so asked me again. I said I wasn't interested. They told me that it paid more money. I became a data science professor. I really liked it and was able to keep a lot of the social data and concerns that made me like sociology through the transition.

Math is sometimes described as a gatekeeper to a lot of things, but it can also be a gateway (hopefully). The question from math folks might be: as a data science faculty, what math do you think is truly essential for your students?

Statistics is obviously core to data science, so I would list those courses first.[1] The class I wish I had taken as an undergraduate is linear algebra. When I got to advanced stats and game theory in grad school, I didn't even know how to read matrix notation, much less do the necessary operations. It kicked my butt for a while as I learned it on my own. It would have been much better to have known it going in.

A positive narrative from the math program's point of view is that some data students end up also falling in love with math, which to be clear, does happen sometimes, but we also know that not every student who comes to data science loves math. Can they still thrive in the field and if so what would that look like? On the flip side, what value does math add to a data scientist's toolbox?

I think that to thrive as a data scientist, you need to like numbers and quantitative logic. You might not love calculus and still thrive in the field, but you can't be numbers averse. A lot of insights come more from thinking through correct or clever comparisons than sophisticated modeling. As models get more complex, however, understanding the math behind them really helps you be aware of their strengths, weaknesses, and limits.

Let's dive deeper into the program building experience you had with data science. In fact, you've been central to building out the data science program here over the years. What's your long-term vision for it? What have been the challenges?

I've been really happy with how the Data Science program has developed over the last few years. Recruiting students was a challenge when we launched as no one had been on campus for a year due to COVID, but we've built momentum since. Going forward, I would like the program to grow a bit more so that we could offer more specific upper-level courses to help students target particular job sectors and keep up with the ever changing world of data analysis.

Math recently launched the new Applied Statistics minor and the majority of the first cohort of minors are data science students. How do you see that fitting into the broader picture given that data science and statistics overlap in some ways although not entirely so?

I think the Applied Statistics minor will be great for Data Science students. Because we're interdisciplinary (meaning we make students take a minor and it counts toward their major credits), we don't have as many statistics requirements in our core curriculum as some data science programs. For students who really want to understand the mathematical underpinnings of what they are doing, the Applied Statistics minor is great. It

[1] He did ask to clarify whether one would consider statistics as part of math, to which the interviewer responded: "I consider everything part of mathematics!"

would set them up very well for graduate school or more mathematically technical jobs.

Math and data science majors seem to increasingly cross paths. How do you see that relationship evolving? Are you happy with sharing the hallway with math people in Dawson?

I think it's great that Math and Data Science are housed in the same space. As has been noted, there is a lot of complementarities and so bumping into Math folks regularly helps us maximize those.

Early on, you took on some intro math courses that weren't officially part of your job description, and by your own account, you said you really enjoyed it. What was that experience like? What aspects of it made it memorable?

I taught MATH 120 and 126 at Nevada State before there were sufficient Data Science classes to fill out my schedule. I was nervous going in as it wasn't my specialty field and I worried that I wouldn't do a good job. Particularly in Math 126. I don't think I ever even took a precalculus course (though I had taken Calculus). They turned out well. I felt I got to look at some things in a new way and the students were happy to go along with that.

Let's flip the script here: What would you say to a math major who is "data-curious" but hasn't made the leap to double major or minor and vice/versa? There are many math majors who end up looking to go in "data" direction when it comes to graduate school or jobs.

I would say take some stats classes and possibly DATA 101 or DATA 210. Both (and especially the former) is more about the logic of data comparison. It shouldn't be a hard class for a math major (especially if they've already taken CS 138) but it will get them thinking about whether data analysis is what they'd like to do.

Outside of Dawson people often lump together math and data science, but what do you notice about the differences between math students and data science students? As in, how they think, approach problems, or even silly stuff like data science students like computer games more (I made that up)? Any such observations?

I don't know enough (non-double) Math majors to answer this question well, but my sense is that the Data Science majors are more utilitarian on average, more choosing the major as a step toward their professional goals. Data Science isn't a common subject in high school and it's fairly new, so I also think a lot more come in with only loose ideas of what it's about.

Wrapping up the "teaching"-side questions, do you have any particularly memorable student work or class project that stuck with you?

One year in DATA 480 (the capstone seminar), half the class was working on sports analytics projects. They varied in their success, but their legacy lives on as we now have a course in the area (DATA 360) as I was so excited about the enthusiasm the students were bringing and the wide variety of data tools they needed to bring together to pursue those projects.

Let's talk future. Actually, let's talk about what's happening now. There is a lot of noise right now about AI and the job market. What advice do you have for students looking to graduate into the somewhat lop-

sided job market? What skills do you think will genuinely matter for data science (and other math-heavy graduates) going forward?

This is the million dollar question right now. I think it will become even more about your ability to define a question, rather than your ability to get a model to run without errors. We've always stressed that, but it becomes an even bigger differentiator as AI tools automate some of the analytic pipeline. You should still be able to do the analysis, though, even if you're going to be using AI. Its results aren't always correct and you need to be able to recognize that.

Future means more immediate for our students. I often hear about graduate school coming from data science students. Are there good options for graduate programs specific to data science out there? If so, would you recommend going into graduate programs?

I generally recommend that students coming out of undergrad get a job first. Graduate school lets you hone a focus, so it helps to be sure of the area that you want to be in before you go and invest that time. Plus, you may be able to get your job to pay for it. UNLV has an online program. UNR has a program. A number of our students have done Georgia Tech's online program after graduation and it looks solid to me.

One last thing: Are you secretely a vampire? You moved from a windowed office to one without windows this semester (and deeper into the math enclave of Dawson).[2]

I just felt that my office was too large with too much light. DAW 222 was open and came preconfigured with a ceiling tile with substantial water damage, so I felt right at home (even if it lacked the actual in-room shower that 216 offered). I try to be upfront about my vampirism. ■

Dr. Tim Malacarne
Associate Professor of Data Science

[2]While fully aware of the unfortunate events last Fall, we still felt like asking him whether he had vampiric tendencies seeking darker spots around campus.

THE SHAPE OF SQUARES: A JOURNEY INTO QUADRATIC FORMS

Dr. Nandita Sahajpal

What do a 17th-century theorem about prime numbers, a 21st-century machine learning algorithm, and the geometry of space-time have in common? All are connected through the mathematics of quadratic forms.

Fig. 1 McGucken Light Cone Spacetime Sculpture, Utah Desert. A visual representation of the light cone in space-time.

EQUATIONS involving squares appear throughout mathematics. At first they look familiar and simple, arising naturally from ideas such as the Pythagorean theorem. Yet these equations lead to surprisingly deep questions. Understanding their solutions connects ideas from number theory, geometry, linear algebra, and topology. The goal of this article is to explore how expressions involving squares lead to the modern theory of quadratic forms and why these objects continue to play an important role in mathematics.

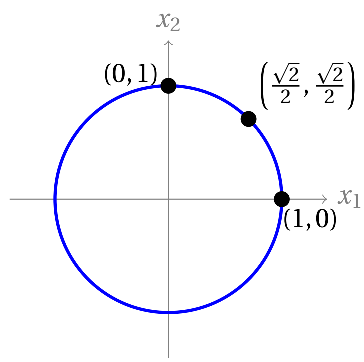

Fig. 2 The solution set of the equation $x_1^2 + x_2^2 = 1$, which forms a circle of radius 1 centered at the origin.

Many geometric objects can be described as the set of solutions to an equation. Suppose we begin with a very simple equation $x_1^2 + x_2^2 = 1$. What does it mean to solve this equation?

A **solution** is simply a pair of real numbers (x_1, x_2) that makes the equation true. For example, the following points are solutions:

$$(1,0),\ (0,1),\ \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right).$$

These are just three examples. In fact, there are **infinitely many** solutions. To understand what all of these solutions look like, it helps to think of (x_1, x_2) not just as two numbers, but as a point in the plane $\mathbb{R}^2$. Viewed this way, the equation $x_1^2 + x_2^2 = 1$ describes all points whose distance from the origin is 1. In other words, the solutions form a circle of radius 1 centered at the origin. Since every point on this circle satisfies the equation, the equation has infinitely many solutions.

Now let us change the equation slightly and see what happens. Consider $x_1^2 - x_2^2 = 0$. At first glance this equation looks very similar to the previous one, but the set of solutions turns out to be completely different. Notice that the expression factors as $x_1^2 - x_2^2 = (x_1 - x_2)(x_1 + x_2)$. So the equation is satisfied whenever $x_1 = x_2$ or $x_1 = -x_2$. These equations describe two lines passing through the origin.

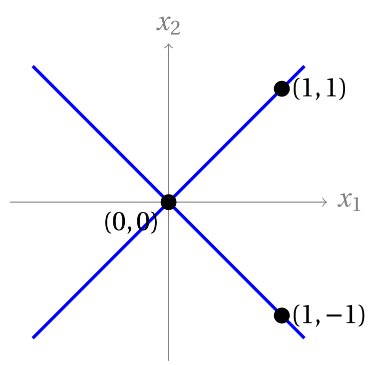

Fig. 3 The solution set of $x_1^2 - x_2^2 = 0$.

Once again there are infinitely many solutions, but the geometric picture is very different.

In the first equation the solutions form a circle. In the second equation the solutions lie on two lines. Two equations that look very similar produce completely different geometric shapes. Why does this happen?

At first glance these equations seem very simple, yet they already hint at a deeper phenomenon: even elementary expressions involving squares can encode surprisingly rich mathematical structure.

What happens if we add another variable and study an equation such as $x_1^2 + x_2^2 - x_3^2 = 0$? Instead of a curve in the plane, we now obtain a surface in three-dimensional space (See Fig. 4).

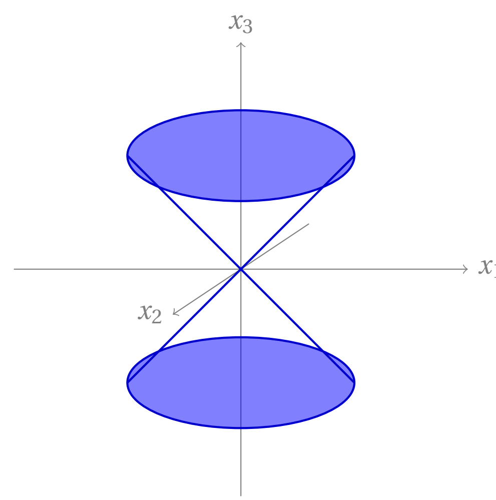

Fig. 4 The quadric surface defined by $x_1^2 + x_2^2 - x_3^2 = 0$ which forms a double cone.

And, what happens if we look at more than one equation at the same time? Suppose that we have a system

$$f(x_1, \ldots, x_n) = 0, \; g(x_1, \ldots, x_n) = 0.$$

What do the solutions look like then? Geometrically, we are asking how the shapes defined by these equations intersect. Questions like these lie at the heart of the study of quadratic forms. Along the way we will encounter ideas from several areas of mathematics. Linear algebra helps us understand quadratic expressions through matrices and eigenvalues. Geometry helps us visualize the shapes defined by these equations. Topology helps us understand how these shapes are connected and how they intersect.

What begins as a simple equation involving squares eventually leads to deep questions about the structure of space itself. This is one of the beautiful aspects of pure mathematics: ideas arising from simple algebraic expressions often reveal unexpected connections across different areas of mathematics and frequently appear in applications ranging from physics and optimization to data science and machine learning.

The goal of this article is to explore some of these ideas through the study of quadratic forms.

Where Do Quadratic Forms Come From?

The simple examples we saw in the previous section raise a natural question: how long have mathematicians been studying equations involving squares?

In fact, questions of this kind have fascinated mathematicians for centuries. Long before the language of quadratic forms existed, mathematicians were already studying equations involving squares.

One of the oldest and most famous examples is the equation $x_1^2 + x_2^2 = x_3^2$. This is the equation behind the Pythagorean theorem. Integer solutions (x_1, x_2, x_3) are known as **Pythagorean triples**. The most familiar example is $(3, 4, 5)$. But this is only the beginning. There are infinitely many such triples: $(5, 12, 13)$, $(8, 15, 17)$, $(7, 24, 25)$.

Naturally, mathematicians began asking deeper questions. Which integers can be written as a sum of two squares? How many ways can a number be represented in this form? Is there a pattern that explains when such representations are possible? One remarkable answer to these questions was discovered by Fermat.

Fermat's Theorem on Sums of Two Squares.

A prime number p can be written in the form

$$p = x_1^2 + x_2^2$$

for integers x_1, x_2 if and only if $p \equiv 1 \pmod 4$. These primes are known as Pythagorean primes and have a unique representation as a sum of two squares.

For example, $5 = 1^2 + 2^2$, $13 = 2^2 + 3^2$, $17 = 1^2 + 4^2$. In contrast, primes such as $3, 7$, and 11 cannot be written as a sum of two squares. Results like this reveal that even very simple-looking equations involving squares can hide surprisingly interesting patterns.

Similar questions appeared in many parts of the world. In 628 AD, an Indian mathematician and astronomer, Brahmagupta, studied equations of the form $x_1^2 - nx_2^2 = c$. In particular, he studied the equation $x_1^2 - nx_2^2 = 1$, where n is a positive integer that is not a perfect square. Finding integer solutions to this equation was far from obvious, yet Brahmagupta developed methods that generate infinitely many solutions. Today this equation is known as **Pell's equation**, and it continues to play an important role in number theory.

By the beginning of the nineteenth century, the study of equations involving squares had become a central topic in mathematics. In 1801, Carl Friedrich Gauss published his famous book *Disquisitiones Arithmeticae*. A large portion of this book is devoted to the systematic study of expressions involving squares of variables, now known as quadratic forms. He introduced powerful ideas for classifying these expressions and understanding when numbers can be represented by them. With Gauss, the study of equations involving squares changed from a collection of isolated problems into a systematic theory. His work laid the foundations for the modern theory of quadratic forms.

Over time mathematicians began to study these expressions not only over the integers but over more general number systems such as the rational numbers, the real numbers and other arbitrary rings and fields. This shift made it possible to use tools from linear algebra to understand quadratic expressions.

Today quadratic forms appear naturally in many areas of mathematics and in many real-world applications. They play an important role in number theory, geometry, optimization, and mathematical physics. In **linear algebra**, they arise naturally when studying symmetric matrices and eigenvalues. In **geometry**, they describe familiar shapes such as ellipses, hyperbolas, and cones. In **physics**, they are used to describe energy and the geometry of space-time. In **optimization**, quadratic forms arise when studying maxima, minima, and quadratic programming problems. In **statistics**, **data science**, and **machine learning**, they appear in covariance matrices, least squares methods, and principal component analysis. Even though quadratic forms may look abstract at first, they repeatedly arise whenever we try to measure shape, distance, variation, or stability.

Did You Know?

Quadratic forms also appear in physics through the geometry of **space-time**. In Euclidean geometry, the **square of the distance** from the origin is given by the quadratic form:

$$d^2 = x_1^2 + x_2^2 + x_3^2.$$

In special relativity, however, “distance" is measured by an interval with mixed signs, such as:

$$s^2 = x_1^2 + x_2^2 + x_3^2 - c^2 t^2.$$

This expression is a quadratic form known as the **Minkowski metric**. Hermann Minkowski showed that Einstein's theory of relativity can be understood geometrically using this quadratic form.

Did You Know?

Quadratic forms also appear in one of the most widely used techniques in science and engineering: the **least squares method**.
When we try to fit a model to data, we often minimize the sum of squared errors

$$(x_1 - a_1)^2 + (x_2 - a_2)^2 + \cdots + (x_n - a_n)^2.$$

This expression is itself a (translated) quadratic form. Finding the best fit amounts to minimizing this quadratic function, which leads naturally to systems of linear equations and eigenvalue problems. Because of this connection, quadratic forms play an important role in statistics, machine learning, and many optimization algorithms.

In the next section we return to the questions raised earlier: what happens when we study quadratic expressions in many variables, and what happens when we study systems of equations involving them?

Quadratic Forms and Linear Algebra

We now return to the kinds of equations we saw earlier, but with a more general point of view. A real **quadratic form** in n variables is a polynomial with real coefficients, that is, $f \in \mathbb{R}[X_1,\dots,X_n]$, in which every term has degree exactly two. In other words, each term involves either the square of a variable or a product of two variables.

Definition: Quadratic Form

A **quadratic form** in n variables over $\mathbb{R}$ is a homogeneous polynomial of degree 2, so it has the form

$$f(X_1,\dots,X_n) = \sum_{i=1}^{n} a_{ii}X_i^2 + \sum_{1\le i<j\le n} a_{ij}X_iX_j,$$

where the coefficients a_{ij} are real numbers.

For example, $f = X_1^2 + X_2^2 - X_3^2$ is a quadratic form in three variables. Another example is $g = X_1^2 + 2X_1X_2 + 3X_2^2$. To evaluate such an expression we substitute numbers for the variables. If $x = (x_1, x_2, x_3) \in \mathbb{R}^3$, then $f(x) = x_1^2 + x_2^2 - x_3^2$. The equation $f(x) = 0$ therefore describes a cone in $\mathbb{R}^3$.

Quadratic forms become especially interesting when we realize that they can be studied using tools from linear algebra.

Matrix Representation of a Quadratic Form

Every quadratic form can be written using matrix notation.
If

$$f(X_1,\dots,X_n) = \sum_{i=1}^{n} a_{ii}X_i^2 + \sum_{1\le i<j\le n} a_{ij}X_iX_j,$$

then there exists a symmetric matrix $A = (a^*_{ij})$ such that $f(x) = x^TAx$, where

$$x = \begin{pmatrix} X_1 \\ \vdots \\ X_n \end{pmatrix}.$$

The entries of A are determined by the coefficients of the quadratic form:

$$a^*_{ii} = a_{ii}, \qquad a^*_{ij} = \frac{1}{2}a_{ij} \quad (i \ne j).$$

In other words, the diagonal entries of A are the coefficients of the square terms, while each cross-term coefficient is split equally between the symmetric positions (i, j) and (j, i).

For example, the quadratic form $g = X_1^2 + 2X_1X_2 + 3X_2^2$ can be written as

$$g(x) = \begin{pmatrix} X_1 & X_2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}.$$

This representation reveals an important connection: studying quadratic forms is closely related to studying **symmetric matrices**.

Did You Know?

The same ideas that appear in the theory of quadratic forms also arise in modern data science. One example is **Principal Component Analysis (PCA)**, a technique used to analyze large datasets. In PCA, a computer studies a covariance matrix and finds its eigenvectors and eigenvalues to determine the directions in which the data varies the most.

Geometrically, this process identifies the principal axes of a multidimensional quadratic form.

One of the central results of linear algebra allows us to diagonalize symmetric matrices.

The Spectral Theorem

Every real symmetric matrix A can be diagonalized by an orthogonal change of coordinates. That is, there exists an orthogonal matrix Q such that

$$Q^T AQ = \begin{pmatrix} d_1 & & 0 \\ & \ddots & \\ 0 & & d_n \end{pmatrix}.$$

Consequently, every quadratic form $f(x) = x^T Ax$ can be written as

$$d_1 X_1^2 + \cdots + d_n X_n^2$$

after a suitable change of variables.

When we apply this idea to quadratic forms, it means that after a suitable change of variables, every quadratic form can be written in the simpler form

$$d_1 X_1^2 + d_2 X_2^2 + \cdots + d_n X_n^2.$$

This representation allows us to understand the geometry of the equation $f(x) = 0$. For instance, if all the coefficients $d_1, \dots, d_n$ are positive, then $f(x) > 0$ for every nonzero vector x. In this case the equation $f(x) = 0$ has only the trivial solution. On the other hand, if some coefficients are positive and others are negative, then the quadratic form can take both positive and negative values. In this situation the equation $f(x) = 0$ often describes interesting geometric objects such as $n-$dimensional cones or hyperboloids.

There are two quantities that help describe this structure. The **rank** of a quadratic form measures how many variables actually appear after diagonalization. In terms of linear algebra, if a quadratic form is represented by a symmetric matrix A, the **rank** of the form is simply the **rank** of A, which equals the number of nonzero eigenvalues of A.

The **signature** records how many of the coefficients d_i are positive and how many are negative after diagonalization. Equivalently, it counts the number of positive and negative eigenvalues of the associated symmetric matrix. By checking the **signature**, we can instantly tell if our “shape" is an ellipsoid, a hyperboloid, or a cone.

These ideas allow us to understand the geometry of a single quadratic equation. But an even richer picture emerges when we study *two* quadratic forms at the same time.

Pairs of Quadratic Forms

Up to this point we have been studying a single quadratic equation $f(x) = 0$. Linear algebra gives us powerful tools for understanding such equations. By diagonalizing the associated matrix, we can determine the basic shape of the geometric object defined by the equation. But mathematics often becomes more interesting when we study more than one equation at the same time.

Suppose we have two real quadratic forms $f, g \in \mathbb{R}[X_1, \dots, X_n]$. Instead of studying the equation $f(x) = 0$ by itself, we now consider the system $f(x) = 0$, $g(x) = 0$, where $x = (x_1, \dots, x_n) \in \mathbb{R}^n$. Geometrically, each equation describes a shape in space. The equation $f(x) = 0$ describes one surface, while $g(x) = 0$ describes another. The question becomes:

> *What do the points look like where these two shapes intersect?*

Sometimes the answer is easy to visualize. For example, in three dimensions, two surfaces might intersect in a curve. In higher dimensions, the geometry becomes harder to picture, but the underlying idea is the same: we are looking for points that satisfy both equations simultaneously. At first, it might seem that if each quadratic form individually has many solutions, then the system should also have many solutions. Surprisingly, this is not always true. For example, consider the quadratic forms

$$f = 2X_1X_2, \qquad g = X_1^2 - X_2^2.$$

Each of these expressions can take both positive and negative values, so each equation by itself has many solutions. However, if we try to solve this system simultaneously, we discover that the only common solution is the trivial one $x = (0,0)$. Examples like this show that understanding when a system of quadratic equations has nontrivial solutions turns out to be an interesting and challenging problem.

One idea that turns out to be extremely useful is to study all possible linear combinations of the two quadratic forms. Given real numbers λ and μ, we can form a new quadratic form

$$\lambda f + \mu g.$$

As λ and μ vary, we obtain an entire family of quadratic forms. This family is called the **pencil of quadratic forms** generated by f and g. Studying this pencil reveals important information about the relationship between the two original equations. In particular, the behavior of the quadratic forms $\lambda f + \mu g$ can tell us whether the system $f(x) = 0$, $g(x) = 0$ has nontrivial solutions.

Theorem

Let f and g be quadratic forms in $n \geq 3$ variables over $\mathbb{R}$.
The system

$$f(x) = 0, \qquad g(x) = 0$$

has a nontrivial real solution if and only if every nontrivial linear combination

$$\lambda f + \mu g$$

is not a definite quadratic form for any real λ, μ that are not both zero.

This result shows that instead of studying two equations separately, it is often useful to examine the entire family of quadratic forms $\lambda f + \mu g$. In the next section we will explore how properties such as **rank** and signature change within this pencil and how these ideas help us understand the geometry of systems of quadratic equations.

What Can the Pencil Tell Us?

In the previous section we introduced the pencil of quadratic forms generated by two quadratic forms f and g:

$$\lambda f + \mu g,$$

where λ and μ are real numbers. Each choice of (λ, μ) produces another quadratic form. One way to think about a pencil of quadratic forms is to imagine gradually blending the two forms f and g. As the parameters λ and μ vary, the quadratic form $\lambda f + \mu g$ changes, and the geometry of the equation changes with it.

Occasionally the quadratic form loses **rank**. These special points reveal important structural information shared by the two original forms. If $f(x) = x^T Ax$ and $g(x) = x^T Bx$ are represented by symmetric matrices A and B, then the quadratic form $\lambda f + \mu g$ corresponds to the matrix $\lambda A + \mu B$. The values of (λ, μ) where the rank drops occur precisely when the determinant

$$\det(\lambda A + \mu B)$$

equals zero. By studying where this happens, we gain insight into whether the system $f(x) = 0$, $g(x) = 0$ has nontrivial solutions.

Why is this useful? The key observation is that properties of the quadratic forms in this pencil often reveal information about this system. In particular, the **rank** and **signature** of the quadratic forms in the pencil play an important role. Recall that after diagonalization, a quadratic form can be written as

$$d_1 X_1^2 + d_2 X_2^2 + \cdots + d_n X_n^2.$$

The **rank** counts how many of the coefficients d_i are nonzero, while the signature records how many are positive and how many are negative. As we vary λ and μ, the quadratic form $\lambda f + \mu g$ changes. Sometimes its **rank** changes, and sometimes the balance between positive and negative coefficients changes as well. These changes are not random. In fact, they contain important information about the geometry of the equations $f(x) = 0$ and $g(x) = 0$.

For example, if a quadratic form has both positive and negative coefficients, then it can take both positive and negative values. This often makes it possible to find nontrivial solutions to the equation $f(x) = 0$. On the other hand, if a quadratic form is positive definite (meaning all the coefficients d_i are positive), then the equation $f(x) = 0$ has only the trivial solution.

By examining how these properties vary within the pencil $\lambda f + \mu g$, we can learn a great deal about whether the system $f(x) = 0$, $g(x) = 0$ admits nontrivial solutions.

Understanding the relationship between pairs of quadratic forms and the behavior of their associated pencil is a central theme in the theory of quadratic forms. Over the real numbers and many other fields, the behavior of pairs of quadratic forms is now fairly well understood (see, for example the work of Davenport, Swinnerton-Dyer, and Heath-Brown). However, the situation becomes much more complicated when we consider systems involving three or more quadratic forms. In these cases the existence of solutions can depend on subtle interactions between the forms and the number of variables involved. Studying these systems and how the behavior of the pencil reflects the geometry of their intersections continues to be an active area of mathematical research.

Looking Ahead

We began with simple equations involving squares and saw how their solutions can be viewed geometrically. Through the language of quadratic forms, tools from linear algebra allow us to study these expressions systematically, while geometry and topology help us understand the shapes and intersections they define. When two quadratic forms are studied together, new questions emerge. By examining the pencil of quadratic forms, $\lambda f + \mu g$, we gain insight into when the system $f(x) = 0$, $g(x) = 0$ has nontrivial solutions. But even understanding two quadratic equations is only the beginning. One can ask similar questions about systems involving three, four, or even more quadratic equations.

Geometrically, each quadratic form defines a surface called a quadric, and solving several such equations simultaneously amounts to studying the intersections of these quadrics. As the number of these forms increases, the geometry of these intersections becomes increasingly difficult to understand. How do the geometric objects defined by these equations intersect? When do such systems have nontrivial solutions? What happens in higher dimensions? How do the properties of the pencil change as λ and μ vary? What new phenomena appear when we study three or more quadratic forms simultaneously? Many of these questions remain only partially understood.

This interplay between simplicity and depth, between the known and the unknown, is one of the things that draws mathematicians to the study of quadratic forms. Problems that begin with equations as simple as sums or differences of squares quickly open the door to deeper ideas, reminding us that even the most elementary expressions may conceal mathematical structures that are still not fully understood.

If the answers were obvious, they would have been found long ago. Instead, these questions continue to challenge us, suggesting that something deeper remains to be discovered. ■

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Challenge Problem

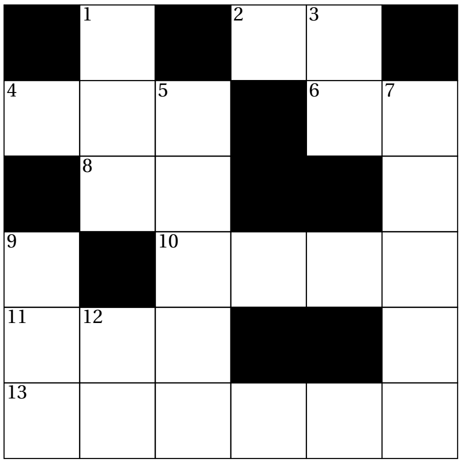
*A **cyclic quadrilateral** is a quadrilateral whose four vertices lie on a single circle. Let P denote the perimeter and A denote the area of a cyclic quadrilateral.*

Problem. *Prove that the perimeter P and area A of a cyclic quadrilateral satisfy*

$$P^2 \geq 16A,$$

with equality if and only if the cyclic quadrilateral is a square.

CROSSWORD CORNER



ACROSS

2. Ceiling function rounds ___
4. Like “fetch”, the limit ___
6. ∊
8. The human-facing side of software
10. Euclid did what? (abbr.)
13. MATH 491 falls on this day for Fall ’26

DOWN

1. Recursive acronym: “___ is not Unix”
3. $\frac{Log(-1)}{i}$
5. This vector owns its transformation
7. “Array you happy this library exists?”
9. Vampiric faculty of data science
11. Things like VI, EMACS, VS CODE, TeXStudio
12. Fortran’s loop

APPLIED STATISTICS

A new interdisciplinary minor

Why Applied Statistics?

Turn your math and data skills into real-world impact. The Applied Statistics Minor gives you the analytical tools to tackle problems across every discipline.

- ✓ Maximize your mathematical reasoning combined with data-analytic skills
- ✓ Expand your statistical toolkit for tackling diverse, complex problems
- ✓ Add a rigorous quantitative dimension to your degree

Core Course Progression

STAT 152 (Introduction to Statistics)
→ APST 311 (Applied Statistics I)
→ APST 312 (Applied Statistics II) & MATH 352 (Probability and Statistics)

STAT 152 and CS 138 are already required for math and data science majors!

Additional Requirements

- ✓ CS 138 (Programming For Data Science)
- ✓ A stats course in your degree program (e.g. PSY 210, STAT 391, DATA 330)

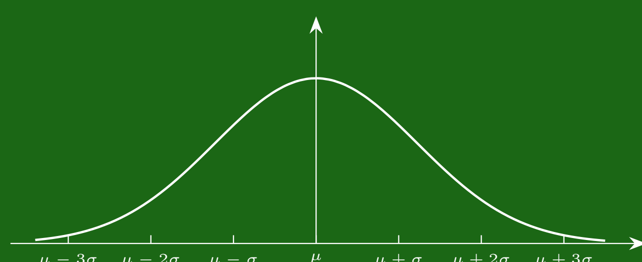
Plan Ahead!

1. Declare the Minor Now!
2. Take STAT 152 ASAP
3. Plan to take APST 311 in Fall 2026 (if you have already taken STAT 152)

Quantitative Skills,

Amplified—

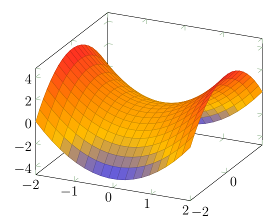
Bring your mathematical expertise to *any discipline*.



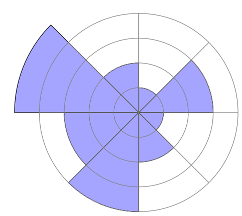
Got Questions?

Get in touch with any of these professors!

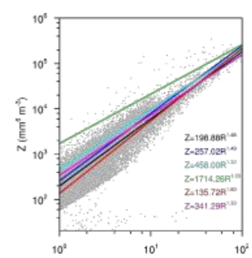
PROF. Sungju Moon
PROF. Krystle Oates
PROF. Aaron Wong



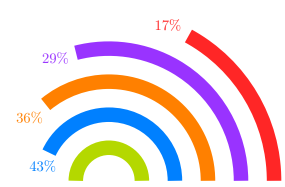
Optimization



PCA/EOF Methods



Regression Analysis



Statistical Inference